# Probability and Analysis on Random Structures and Related Topics

*RIMS Symposium, 8-10 August 2022*

Research Institute for Mathematical Sciences, Kyoto University

**Organisers: David Croydon (Kyoto University), Ryoki Fukushima (University of Tsukuba), Naotaka Kajino (Kyoto University)**

| Monday 8th August | Tuesday 9th August | Wednesday 10th August |
|---|---|---|
| 10:00–10:40 ..... Seiichiro Kusuoka | 09:30–10:10 ....... Shuta Nakajima | 09:30–10:10 ........ Yuki Tokushige |
| 10:50–11:30 ..... Reika Fukuizumi* | 10:20–11:00 ......... Naoki Kubota | 10:20–11:00 ...... Daisuke Shiraishi |
| 11:40–12:20 ....... Masato Hoshino | 11:20–12:00 .... Makoto Nakashima | 11:20–12:00 ...... Takashi Kumagai |
| 14:00–14:50 ...... Stefan Neukamm | 13:30–14:20 ...... Sebastian Andres | 13:30–14:10 ........ Makiko Sasada |
| 15:00–15:25 ...... Yutaka Takeuchi | 14:30–14:55 ........... Takumu Ooi | 14:20–15:00 ....... Hirofumi Osada |
| 15:40–16:20 ............ Akira Sakai | 15:05–15:30 ..... Satomi Watanabe | |
| 16:30–17:10 ... Tomohiro Sasamoto | 15:50–16:30 ...... Ryokichi Tanaka | |
| | 16:40–17:20 .......... Stefan Junk* | |

$*$ = online presentation

Sebastian Andres ........ *First passage percolation with long-range correlations*

Reika Fukuizumi ........ *Pattern formation in stochastic Swift-Hohenberg equation*

Masato Hoshino ........ *A semigroup approach to the multi-level Schauder estimate*

Stefan Junk ........ *Fluctuations for the partition function of directed polymers beyond the $L^2$ phase*

Naoki Kubota ........ *Strict comparison for the Lyapunov exponents of the simple random walk in random potentials*

Takashi Kumagai ........ *Periodic homogenization of jump-type processes with drifts*

Seiichiro Kusuoka ........ *Stochastic quantization associated with the* $\exp(\Phi)_2$*-quantum field model*

Shuta Nakajima ........ *A variational formula for large deviations in first-passage percolation under tail estimates*

Makoto Nakashima ........ *A remark of the free energy for some disordered systems*

Stefan Neukamm ........ *Large-scale Schauder estimate and applications to error estimates in stochastic homogenization*

Takumu Ooi ........ *Convergence of time-changed* $\alpha$*-stable processes by GMC*

Hirofumi Osada ........ *The sub-diffusivity and diffusivity of tagged particles for interacting Brownian particles*

Akira Sakai ........ *Spread-out limit of the critical points for various statistical-mechanics models*

Makiko Sasada ........ *On the stationary solutions of random polymer models and their zero-temperature limits*

Tomohiro Sasamoto ........ *Fluctuations of the log gamma polymer in half space*

Daisuke Shiraishi ........ *Exceptional sets for simple random walks*

Yutaka Takeuchi ........ *Homogenization of reflecting diffusions in a continuum percolation cluster*

Ryokichi Tanaka ........ *Quasi-conformal measures and harmonic measures for hyperbolic groups: multifractal analysis*

Yuki Tokushige ........ *On scaling limits of SRWs on long-range percolation clusters*

Satomi Watanabe ........ *Heat kernel fluctuations for the three-dimensional uniform spanning tree*

`https://www.kurims.kyoto-u.ac.jp/~croydon/ProbAnalRandStruRelaTopi.html`

**Sebastian Andres (University of Manchester)**

*First passage percolation with long-range correlations*

In this talk we consider first passage percolation (FPP) with passage times generated by a general class of models with long-range correlations, including discrete Gaussian free fields as a prominent example. We will discuss conditions under which the associated time constant is positive and the FPP distance is comparable to the Euclidean distance. We will also present two applications to random conductance models (RCM) with possibly unbounded and strongly correlated conductances, namely a Gaussian heat kernel upper bound for RCMs with a general class of speed measures, and an exponential decay estimate for the Green function of RCMs with random killing measures. This talk is based on a joined work with Alexis Prévost (Cambridge).

**Reika Fukuizumi (Tohoku University)**

*Pattern formation in stochastic Swift-Hohenberg equation*

We study a phenomenological model for pattern formations in the electroconvection, and the effect of noise on the pattern. As such model we consider an anisotropic Swift-Hohenberg equation adding an additive noise. We prove the existence of a global solution of that equation on the two dimensional torus. In addition, inserting a scaling parameter, we consider the equation on a large domain near its change of stability, we observe numerically that, under the appropriate scaling, its solutions can be approximated by a periodic wave, which is modulated by the solutions to a stochastic Ginzburg-Landau equation. This is a joint work with Yueyuan Gao, Guido Schneider, and Motomitsu Takahashi.

**Masato Hoshino (Osaka University)**

*A semigroup approach to the multi-level Schauder estimate*

We introduce a semigroup approach to prove the reconstruction theorem and the multi-level Schauder estimate. These two theorems play important roles in the analytic part of the theory of regularity structures. The first one is an application of the method by Otto and Weber, but the second one is new. This approach allows us to treat semilinear/quasilinear parabolic SPDEs with variable coefficients. This talk is based on a joint work with Ismaël Bailleul (Université Rennes 1) and an ongoing work with Bailleul and Seiichiro Kusuoka (Kyoto University).

**Stefan Junk (Tohoku University)**

*Fluctuations for the partition function of directed polymers beyond the $L^2$ phase*

The directed polymer model describes random paths under the influence of a space-time random environment. We consider the high-temperature, weak disorder regime in spatial dimension $d \geq 3$, where it is known that the polymer obeys a central limit theorem. Many author focus on a (strict) sub-regime of the weak disorder phase, which is characterized by $L^2$-boundedness of an associated martingale $(W_n)_n$. In contrast, we study the situation with unbounded second moments, specifically the spatial correlations of the field $(W_n^x)_x$, where the superscript $x$ indicates the starting location of the polymer. In the case of a bounded environment, we show that a suitably re-centered spatial average over a set of diameter $n^{1/2}$ converges to zero at rate $n^{-a+o(1)}$, with an explicit exponent that is different from the corresponding exponent in the $L^2$-bounded case.

**Naoki Kubota (Nihon University)**

*Strict comparison for the Lyapunov exponents of the simple random walk in random potentials*

We consider the simple random walk in i.i.d. nonnegative random potentials on the lattice. In this model, the so-called Lyapunov exponent describes the "cost" paid by the walker for traveling from the origin to a remote location in a landscape of potential. The Lyapunov exponent is related to not only the traveling cost but also the large deviation principle. In fact, the rate function is represented by the Lyapunov exponent. Hence, it is important to study the properties of the Lyapunov exponent, but there are still many unknown aspects. In particular, there are a few results on the comparison between Lyapunov exponents for different laws of potential. We focus on this topic, and prove the "strict" comparison between Lyapunov exponents with different laws in some sense.

**Takashi Kumagai (Waseda University)**

*Periodic homogenization of jump-type processes with drifts*

We study homogenization problem for strong Markov processes on $\mathbb{R}^d$ having infinitesimal generators

$$\mathcal{L}f(x) = \int_{\mathbb{R}^d} \big(f(x+z) - f(x) - \langle \nabla f(x), z\rangle \mathbb{1}_{\{|z|\le 1\}}\big)\, k(x,z)\,\Pi(dz) + \langle b(x), \nabla f(x)\rangle,\ \ f \in C_b^2(\mathbb{R}^d)$$

in periodic media, where $\int_{\mathbb{R}^d}(1 \wedge |z|^2)\,\Pi(dz) < \infty$ and $\Pi$ can be singular with respect to the Lebesgue measure. Under a proper scaling, we show the scaled processes converge weakly to Lévy processes on $\mathbb{R}^d$. In particular, we completely characterize the limiting processes when $b(x)$ is bounded continuous, $k(x,z)$ is non-negative bounded continuous and 1-periodic both for $x$ and $z$, and in spherical coordinate

$$\mathbb{1}_{\{|z|>1\}}\,\Pi(dz) = \mathbb{1}_{\{r>1\}}\varrho_0(d\theta)\,\frac{dr}{r^{1+\alpha}}$$

with $\alpha \in (0,\infty)$ and $\varrho_0$ being any finite measure on the unit sphere. Different scaling limits appear depending on the values of $\alpha$. This talk is based on joint work with Xin Chen, Zhen-Qing Chen and Jian Wang (Ann. Probab. 2021).

**Seiichiro Kusuoka (Kyoto University)**

*Stochastic quantization associated with the* $\exp(\Phi)_2$*-quantum field model*

We consider the stochastic quantization equations of the quantum field model with the exponential interactions on the two-dimensional torus. Under the full $L^1$-regime, we construct the unique time-global solution in the sense of singular stochastic partial differential equations, the stationary solution, and the Dirichlet form. Moreover, we obtain the identification of the processes obtained by the methods. This talk is based on jointworks with Masato Hoshino and Hiroshi Kawabi.

**Shuta Nakajima (Meiji University)**

*A variational formula for large deviations in first-passage percolation under tail estimates*

We consider the first-passage percolation with identical and independent weight distributions. In this talk, we discuss the upper tail large deviations under a tail assumption on the distributions. We prove that the corresponding rate function is described by the so-called discrete $p$-capacity, and we study its asymptotic. The talk is based on joint work with Clement Cosco and Florian Schweiger.

**Makoto Nakashima (Nagoya University)**

*A remark of the free energy for some disordered systems*

In this talk, we focus on disordered pinning models and directed polymers in random environment(DPRE). It is known that disordered pinning models have the scaling limits in an intermediate regime when the distribution of the underlying renewal process has a polynomial tail with exponent $\alpha \in (\frac{1}{2}, 1)$. The same phenomena happens for $1 + 1$ dimensional directed polymers in random environment. Moreover, it has been proved that under a certain technical condition, their free energies have a universal asymptotic behavior in the weak coupling regime which is explicitly given in terms of the scaling limits. Recently, the same asymptotic behavior is obtained without the technical condition.

**Stefan Neukamm (TU Dresden)**

*Large-scale Schauder estimate and applications to error estimates in stochastic homogenization*

Since the seminal work by Avellaneda and Lin it is known that elliptic operators with periodic coefficients feature – thanks to homogenization – the same regularity theory on large scales as the Laplacian. In the talk we extend this observation to elliptic systems with stationary and ergodic, random coefficients. In particular, we shall discuss large scale $C^{1,\alpha}$ estimates for such systems. Next, we consider elliptic equations with correlated stationary coefficients. Using the large scale regularity theory, we establish error estimates with optimal scaling for the two-scale expansion of stochastic homogenization.

**Takumu Ooi (Kyoto University)**

*Convergence of time-changed $\alpha$-stable processes by GMC*

We consider Gaussian fields whose covariance kernels are 1-order green functions of $\alpha$-stable processes on $\mathbb{R}^d$ and their Gaussian multiplicative chaos (GMC). It is known that $\alpha$-stable processes converge weakly with Skorokhod's topology as $\alpha \to d$ and, by using the result of [Shamov, 2016], GMC converges weakly with the weak topology. In this talk, we will consider the convergence of the time-changed processes of $\alpha$-stable processes by GMC, and I will talk about results to date (mainly results of the tightness).

**Hirofumi Osada (Chubu University)**

*The sub-diffusivity and diffusivity of tagged particles for interacting Brownian particles*

The celebrated Nash theory for symmetric diffusions clarifies the relation of the time decay of the transition probability density of diffusion with translation-invariant measure and the Nash inequality. If the Nash inequality holds, then the diffusion is diffusive. However, there exists no analogy for infinite-dimensional space.

In this talk, we present a criterion for sub-diffusivity of tagged particles of the stochastic dynamics associated with the translation-invariant random point fields. For this we introduce the concept of decomposability for random point fields. We prove that the tagged particles are sub-diffusive if the random point field is decomposable. The decomposability follows from the number rigidity of random point fields. This result implies the tagged particles of the Ginibre interacting Brownian motion and the diffusions related to the planner Gaussian analytic function are sub-diffusive. We also show a sufficient condition of diffusivity in terms of reduced Palm measures.

**Akira Sakai (Hokkaido University)**

*Spread-out limit of the critical points for various statistical-mechanics models*

Self-avoiding walk and lattice trees/animals are statistical-mechanics models for (linear or branched) polymers in a good solvent. We define these models in $\mathbb{Z}^d$ with the spread-out edge set $\{\{x,y\}\subset\mathbb{Z}^d : 0<\|x-y\|\le L\}$. The statistical properties of each model are encoded in the generating function with fugacity $p/\Delta$, where $\Delta=O(L^d)$ is the degree of a vertex. We are interested in the spread-out limit ($L\uparrow\infty$) of the model-dependent critical point $p_c$.

With Remco van der Hofstad, I was able to show in [2] that $p_c=1+CL^{-d}+O(L^{-d-1})$ for self-avoiding walk, percolation, oriented percolation (in $d+1$ space-time dimensions) and the contact process above their respective upper-critical dimension $d_c$ (e.g., $=4$ for self-avoiding walk), where the model-dependent constant $C$ can be represented in terms of continuous-space random walks:

$$C=\begin{cases}\displaystyle\sum_{n=2}^{\infty}U^{*n}(o) & \text{[self-avoiding walk, the contact process]},\\ \displaystyle\frac{1}{2}\sum_{n=2}^{\infty}U^{*2n}(o) & \text{[oriented percolation]},\\ \displaystyle U^{*2}(o)+\sum_{n=3}^{\infty}\frac{n+1}{2}U^{*n}(o) & \text{[percolation]},\end{cases}$$

where $U^{*n}$ is the $n$-fold convolution in $\mathbb{R}^d$ of the uniform probability distribution $U$ on $\{x\in\mathbb{R}^d : \|x\|\le 1\}$. (A similar result was proven in [1] for long-range oriented percolation in spatial dimensions $d>2(\alpha\wedge 2)$, where the bond-occupation probability for an oriented edge $((x,t),(y,t+1))$ is proportional to $(|x-y|\vee L)^{-d-\alpha}$ with $\alpha>0$.) Recently, Noe Kawamoto and I were able to show in [3] that $p_c=1/e+CL^{-d}+O(L^{-d-1})$ for lattice trees (LT) and lattice animals (LA) above the common upper-critical dimension $d_c=8$, where

$$C_{\mathrm{LT}}=\sum_{n=2}^{\infty}\frac{n+1}{2e}U^{*n}(o),\qquad C_{\mathrm{LA}}=C_{\mathrm{LT}}-\frac{1}{2e^2}\sum_{n=3}^{\infty}U^{*n}(o).$$

At the workshop, I will explain the key ideas for the proof of the above results and potential future work on those models on random conductors.

[1] L.-C. Chen and A. Sakai. Critical behavior and the limit distribution for long-range oriented percolation. I. *Probab. Theory Relat. Fields*, **142** (2008): 151–188.

[2] R.v.d. Hofstad and A. Sakai. Critical points for spread-out self-avoiding walk, percolation and the contact process above the upper critical dimensions. *Probab. Theory Relat. Fields*, **132** (2005): 438–470.

[3] N. Kawamoto and A. Sakai. Spread-out limit of the critical points for lattice trees and lattice animals in dimensions $d > 8$. Preprint: arXiv:2205.09451.

**Makiko Sasada (University of Tokyo)**

*On the stationary solutions of random polymer models and their zero-temperature limits*

In this talk, I will discuss stationary measures for positive-temperature random polymer models and their zero-temperature limits. In particular, we derive new stationary measures for the zero-temperature limit of the beta random polymer, that has been called the river delta model. To do this, we apply techniques developed for understanding the stationary measures of the corresponding positive-temperature random polymer models, highlighting how the maps underlying these systems can each be reduced to one of two basic bijections, and that through an 'independence preservation' property, these bijections characterize the associated stationary measures. We then derive similar descriptions for the corresponding zero-temperature maps, whereby each is written in terms of one of two bijections. I will also show that our systematic approach helps clarify why particular stationary measures arise for the different random polymer models considered, and the connections between them. This talk is based on the joint work with David Croydon.

**Tomohiro Sasamoto (Tokyo Institute of Technology)**

*Fluctuations of the log gamma polymer in half space*

Log gamma polymer, introduced by Seppalainen in 2009, is one of the most well studied model of a directed polymer in random media. It has a natural statistical mechanical interpretation at positive temperature and admits exact analysis. While previous studies mostly focus on the whole space version, there have been much less results for its half-space version.

Recently we discovered a direct connection between solvable models in the KPZ class and free fermionic models at positive temperature (or determinant point processes). The key ingredient in our theory is a new identity between marginals of the q-Whittaker measure and the periodic Schur measure, which is proved in bijective fashion by substantially generalizing the RSK algorithm. Once the connection is established, subsequent analysis becomes rather standard and it allows to study various half-space models by taking certain limits. In this talk we present a Fredholm Pfaffian formula for log gamma polymer in half-space and establish the limit theorems.

The talk is based on collaborations with Takashi Imamura and Matteo Mucciconi [1].

[1] T. Imamura, M. Mucciconi, T. Sasamoto, Solvable models in the KPZ class: approach through periodic and free boundary Schur measure, arXiv: 2204.0842.

**Daisuke Shiraishi (Kyoto University)**

*Exceptional sets for simple random walks*

I will discuss several exceptional sets for simple random walk (SRW) paths: cut points, loop-erased random walk and outer boundary of 2D SRW. I will show that the renormalized counting measure for these sets converges as the lattice spacing goes to zero. This is joint work with Yifan Gao, Xinyi Li and Petr Panov.

**Yutaka Takeuchi (Keio University)**

*Homogenization of reflecting diffusions in a continuum percolation cluster*

Homogenization is one of the important problems of random media. After the 2000s, many researchers showed quenched results. For instance, quenched invariance principle for random walks in random conductance models and diffusions in random environments were shown. In these settings, quenched local central limit theorems are also considered. In this talk, we consider reflecting diffusions in a continuum percolation cluster built over stationary ergodic point processes. Under some geometrical conditions, we obtain a quenched invariance principle and a local central limit theorem.

**Ryokichi Tanaka (Kyoto University)**

*Quasi-conformal measures and harmonic measures for hyperbolic groups: multifractal analysis*

We describe the multifractal structure of measures at infinity for hyperbolic groups. Joint work with Steve Cantrell (Chicago).

**Yuki Tokushige (Kyushu University)**

*On scaling limits of SRWs on long-range percolation clusters*

In this talk, we will discuss a Long-Range Percolation (LRP) on $\mathbb{Z}^d$, which is a variant of the classical nearest-neighbor Bernoulli percolation. In particular, we consider a problem concerning a SRW on the infinite cluster of a LRP. In the paper published in 2013, Crawford and Sly proved that a SRW on the infinite cluster scales to $\alpha$-stable process. However, they completed the proof only when $\alpha \in (0, 1)$. It is because the remaining case involves a technical problem to do with a control of short jumps of a SRW, which is related to the fact that sample paths of $\alpha$-stable process have infinite variations for $\alpha \in [1, 2)$.

In this talk, we will give a brief overview of this subject and explain our idea to overcome the technical issue depicted above and complete the proof of the case $\alpha \in [1, 2)$. This talk is based on a joint work with Noam Berger (Technical University of Munich).

**Satomi Watanabe (Kyoto University)**

*Heat kernel fluctuations for the three-dimensional uniform spanning tree*

Heat kernel fluctuations have been observed in some random walks in random environments. In this talk, I will consider the three-dimensional uniform spanning tree (UST) as a random environment. I will first show the occurrence of log-logarithmic fluctuations around the leading order for the volume of intrinsic balls in the UST. This is then used to obtain similar fluctuations for the quenched heat kernel.